Abstract—The fuzzy inference system (FIS) is useful for developing an improved Risk Priority Number (RPN) model for risk evaluation in failure mode and effect analysis (FMEA). A general FIS_RPN model considers three risk factors, i.e., severity, occurrence, and detection, as the inputs and produces an FIS_RPN score as the output. At present, there are two issues pertaining to practical implementation of classical FIS_RPN models as follows: 1) the fulfillment of the monotonicity property between the FIS_RPN score (output) and the risk factors (inputs); and 2) difficulty in obtaining a complete and monotone fuzzy rule base. The aim of this paper is to propose a new analytical interval FIS_RPN model to solve the aforementioned issues. Specifically, the incomplete and potentially nonmonotone fuzzy rules provided by FMEA users are transformed into a set of interval-valued fuzzy rules in order to produce an interval FIS_RPN model. The interval FIS_RPN model aggregates a set of risk ratings and produces a risk interval, which is useful for risk evaluation and prioritization. Properties of the proposed interval FIS_RPN model are analyzed mathematically. An FMEA procedure that incorporates the proposed interval FIS_RPN model is devised. A case study with real information from a semiconductor company is conducted to evaluate the usefulness of the proposed model. The experimental results indicate that the interval FIS_RPN model is capable of appropriately rank the failure modes, even when the fuzzy rules provided by FMEA users are incomplete and nonmonotone.

Index Terms—Failure mode and effect analysis (FMEA), fuzzy inference systems (FISs), interval approach, monotonicity property, risk analysis.

I. INTRODUCTION

FAILURE mode and effect analysis (FMEA) is a useful and well-known risk prioritization and analysis methodology for a wide range of industries, e.g., semiconductor, aerospace, medical, and automotive [1]. The main goal of FMEA is to define, identify, prioritize, and eliminate known and/or potential failures prior to their occurrences in a product, system, process, and/or service, thereby assuring safety, reliability, and quality [1], [2]. Conventional FMEA adopts the Risk Priority Number (RPN) model for decision making. Typically, the RPN score is determined by multiplying three risk factors solicited from FMEA users, i.e., severity (Sev), occurrence (Occ), and detection (Det). To improve the conventional RPN model, a number of alternatives have been proposed [3], e.g., multicriteria decision making (MCDM), mathematical programming, and artificial intelligence (AI).

The fuzzy inference system (FIS), which is classified as an AI-based method [3], can be used to replace the conventional RPN model in aggregating the Sev, Occ, and Det ratings, resulting in an FIS_RPN model. Specifically, the relationship between the inputs (Sev, Occ, and Det ratings) and the output (FIS_RPN score) is formulated as a set of fuzzy rules. Examples of successful industrial applications of FIS_RPN models include nuclear [5], agriculture [6], marine [7], and automotive [8]. In general, an FIS_RPN model is preferred to the conventional RPN model for the following reasons: 1) it allows the customization of the RPN model based on an expert’s knowledge provided in the form of fuzzy ”IF–THEN” rules [6], [10]; 2) it allows a nonlinear relationship between the FIS_RPN score and the input risk factors to be formulated [4]; 3) it is robust against uncertainty and vagueness due to fuzzy information processing [7], [10]; and 4) it captures the three input risk factors qualitatively, instead of quantitatively, by using fuzzy logic [4].

Nevertheless, there are a number of issues related to the effective implementation of existing FIS_RPN models, which are hereafter known as classical FIS_RPN models, as follows: 1) the fulfillment of the monotonicity property between the FIS_RPN score and its input risk factors (see Definition 5) [11]; 2) difficulty in obtaining complete fuzzy rules [11]; and 3) the obtained fuzzy rules can be nonmonotone, although they should be monotone. The first issue indicates that it is important for a usable and useful FIS_RPN model to fulfill the monotonicity property in order to ensure the validity of the generated FIS_RPN scores [11]. In our previous work [11], a set of sufficient conditions [12] has been derived and adopted as the governing equation for modeling an FIS_RPN model. The sufficient conditions suggest that a complete and monotone fuzzy rule set is required (see Theorem 1) [11]. However, the second issue indicates that, in practice, it is often not feasible to gather a set of complete fuzzy rules, which is tedious and time consuming [7], [11]. In [9], a guided rule reduction system was proposed to tackle this problem. The use of the monotonicity-preserving analogical reasoning (AR) [13] technique and a two-stage framework [11] to approximate the unknown fuzzy rules to form a complete fuzzy rule base was demonstrated. The use of the MCDM approaches [3], [14] or other fuzzy-related
operators [15]–[17], instead of a fuzzy rule base, to avoid the requirement of a complete fuzzy rule set was also proposed.

The third issue suggests that the obtained fuzzy rules can be nonmonotone, although they should be monotone in practice, due to noise. In this paper, noise is perceived as judgment errors from domain experts that can lead to a nonmonotone fuzzy rule. To the best of our knowledge, the third issue has yet to be fully addressed in literature, either in general fuzzy modeling or specific FIS_RPN modeling.

In our previous work on FIS_RPN models [11], the obtained fuzzy rules were assumed to be incomplete but monotone. Instead of using the monotonicity-preserving AR technique to approximate the unknown and to obtain a complete rule set [11], [13], we keep all the available fuzzy rules (potentially nonmonotone and/or incomplete fuzzy rules) as they are, and we transform them into a set of interval-valued fuzzy rules in this paper. As such, an interval FIS_RPN model is formulated. This constitutes an analytical method to tackle the three aforementioned issues related to classical FIS_RPN models. In addition, a new ordering method is introduced for the proposed interval FIS_RPN model. The new ordering method allows all failure modes to be ordered and/or corrective actions to be prioritized, even when the fuzzy rule base is incomplete and potentially nonmonotone.

The contributions of this paper are threefold: 1) an analytical interval FIS_RPN model for handling incomplete and potentially nonmonotone fuzzy rules; 2) the mathematical analysis of the proposed interval FIS_RPN model; and 3) a new ordering method for failure mode ordering and/or corrective action prioritization. The applicability of the proposed approach is demonstrated with real FMEA information from a semiconductor manufacturing plant.

The organization of this paper is as follows. In Section II, a numerical example is used to explain the proposed approach. In Section III, the backgrounds of the conventional RPN and classical FIS_RPN models are described. In Section IV, our proposed approach is explained in detail. In Section V, the experimental results are presented, analyzed, and discussed. Concluding remarks and suggestions for further work are presented in Section VI.

II. NUMERICAL EXAMPLE

A. Benchmark Information From [18]

An example is first presented to explain our proposed approach. The fuzzy risk matrix of an FIS-based risk model from the work in [18] is considered. The FIS-based risk model is a two-input function, i.e., Sev and frequency (in a log scale) (Fre). It produces a risk index, i.e., risk index = f(sev, fre).

Assume that sev ∈ Sev = [1, 5], fre ∈ Fre = [10^−8, 10^7], and risk index = [0, 5]. Fig. 1 shows that Sev and Fre are partitioned into a number of fuzzy sets. The fuzzy sets of Sev, Fre are denoted as μ Sev (sev) with a linguistic term, i.e., A Sev and μ Fre (fre) with a linguistic term, i.e., A Fre, respectively, where n Sev = 1, ..., 5, and n Fre = 1, ..., 7. The linguistic terms of the fuzzy sets for risk index are denoted as A, TA, TNA, and NA, with fuzzy singleton [19] of 1, 2, 3, and 4, respectively. A set of fuzzy rules is used to describe the relation among Sev, Fre, and risk index, i.e., “If Sev is A Sev and Fre is A Fre Then risk index is b Sev,Fre,” where b Sev,Fre ∈ [1, 2, 3, 4].

Fig. 1. (a) Membership functions of Sev (A Sev; negligible; A Sev; low; A Sev; moderate; A Sev; high; A Sev; catastrophic). (b) Membership functions of Fre (A Fre; remote; A Fre; unlikely; A Fre; low; A Fre; medium; A Fre; high; A Fre; very high). (a) Sev. (b) Fre (log 10 (\(10^{-8}\))).

For notational simplicity, fuzzy rules are written as \(R_{n Sev,n Fre}^{n Sev,n Fre} : A_{n Sev,n Fre}^{n Sev,n Fre} \rightarrow b_{n Sev,n Fre}^{n Sev,n Fre}\). Using the grid partition strategy [20], a total of 5 × 7 = 35 fuzzy rules are required to form a complete fuzzy rule set.

Fig. 2(a) illustrates the "standard" fuzzy risk matrix in [18], i.e., a set of complete and monotone fuzzy rules. As an example, consider R1.1 : A1.1 → b1.1, b1.1 = 1. Here, the zero-order Sugeno FIS model [19] is used. It is important to maintain the monotonicity property of the FIS-based risk model. The monotonicity property suggests that, when sev and/or fre increases, risk index should not decrease. According to the works in [11] and [12], maintaining the monotonicity property requires complete and monotone fuzzy rules. In this example, monotone fuzzy rules suggest the following conditions: 1) \(b_{n Sev,n Fre}^{n Sev,n Fre} \leq b_{n Sev,n Fre+1,n Fre}^{n Sev,n Fre+1}\), where \(n_{Sev} = 1, \ldots, 4\), and \(n_{Fre} = 1, \ldots, 7\); and 2) \(b_{n Sev,n Fre}^{n Sev,n Fre} \leq b_{n Sev+1,n Fre}^{n Sev+1,n Fre}\), where \(n_{Sev} = 1, \ldots, 5\), and \(n_{Fre} = 1, \ldots, 6\). The fuzzy rules are denoted as nonmonotone if either one of the conditions is violated.

A complete fuzzy rule set suggests that all \(b_{n Sev,n Fre}^{n Sev,n Fre}\), where \(n_{Sev} = 1, \ldots, 5\) and \(n_{Fre} = 1, \ldots, 7\), should be known and provided. The fuzzy rules are denoted as incomplete if not all \(b_{n Sev,n Fre}^{n Sev,n Fre}\) are provided. Using the information in Fig. 2(a), a
The known fuzzy rule, e.g., interval, i.e., $b^3, l, b^3, l$ is the region in the exemplified fuzzy rule matrix that contains a rule in which its $b_{n_{Frec}}$ should be lower than or equal to $(must not be higher than) b^3, l$, i.e., $A_{n_{Frec}} \leq 3, n_{Frec} \leq 4$. Similarly, the upper region of $A_{n_{Frec}}$ is the region that contains a rule in which its $b_{n_{Frec}}$ should be higher than or equal to $(must not be lower than) b^3, l$, i.e., $A_{n_{Frec}} \leq 3, n_{Frec} \leq 4$. Note that $A_{n_{Frec}}$ and $A_{n_{Frec}}$ are shaded with light gray and dark gray in Fig. 3(a), respectively.

Our proposed approach attempts to transform nonmonotone and/or incomplete fuzzy rules to interval-valued fuzzy rules, i.e., $[b_{n_{Frec}}, b_{n_{Frec}}]$, where $l_{Frec} \in n_{Frec}$, and $l_{Frec} \in n_{Frec}$. It is a two-step approach. In the first step, $b_{n_{Frec}}$ is transformed to a monotone interval, which is denoted as $[b_{n_{Frec}}, b_{n_{Frec}}]$. As such, $b_{n_{Frec}}$ is obtained using $b_{n_{Frec}} = \max(b_{n_{Frec}}, b_{n_{Frec}})$, and $b_{n_{Frec}}$ is obtained using $b_{n_{Frec}} = \min(b_{n_{Frec}}, b_{n_{Frec}})$. The known fuzzy rule, e.g., $b^3, l$, is transformed to a monotone interval, i.e., $[b^3, l, b^3, l]$. As such, $b^3, l$ is obtained using $b^3, l = \max(b^3, l, b^3, l)$, and $b^3, l$ is obtained using $b^3, l = \min(b^3, l, b^3, l)$.

B. With Proposed Approach

The proposed approach is used to solve the aforementioned problem. To facilitate further explanation, two notations pertaining to a fuzzy rule, i.e., the lower and upper regions, are introduced. A fuzzy rule, i.e., $A_{n_{Frec}} \rightarrow b_{n_{Frec}}$, where $3 \in n_{Frec}$ and $4 \in n_{Frec}$, is exemplified. The lower and upper regions of $A_{n_{Frec}}$ are denoted as $A_{n_{Frec}}$ and $A_{n_{Frec}}$, respectively. The lower region of $A_{n_{Frec}}$ is the region in the exemplified fuzzy rule matrix that contains a rule in which its $b_{n_{Frec}}$ should be lower than or equal to $(must not be higher than) b^3, l$, i.e., $A_{n_{Frec}} \leq 3, n_{Frec} \leq 4$. Similarly, the upper region of $A_{n_{Frec}}$ is the region that contains a rule in which its $b_{n_{Frec}}$ should be higher than or equal to $(must not be lower than) b^3, l$, i.e., $A_{n_{Frec}} \leq 3, n_{Frec} \leq 4$. Note that $A_{n_{Frec}}$ and $A_{n_{Frec}}$ are shaded with light gray and dark gray in Fig. 3(a), respectively.

C. Remarks

Studies on handling incomplete fuzzy rules have been reported in literature, e.g., fuzzy rule interpolation [21], [22], AR [13], and a completion algorithm [23]. Such issue is also known as the “tomato classification problem” [24], in which no valid conclusions can be inferred for an observation that falls within the “gap.” The existing methods in literature attempt to approximate fuzzy rules pertaining to such scenario. Recently, another attempt to solve this problem has focused on the monotonicity property in FIS modeling, e.g., some monotonicity theorems are derived for the Sugeno FIS model in [12].

Based on current literature, research on handling incomplete fuzzy rules in FIS models that require the monotonicity property is relatively new. The problem becomes more complicated when the fuzzy rules are incomplete and nonmonotone, as demonstrated with the two-input example. The proposed method in this paper provides a solution to handle issues
pertaining to incomplete and/or nonmonotone fuzzy rules for the risk evaluation and prioritization problem in FMEA. Such problem is more challenging as three input ratings are involved in FIS modeling.

III. FMEA REVISITED

A. Sev, Occ, Det, and RPN

Traditionally, the risk of a failure mode is determined by computing its RPN score. The conventional RPN model considers Sev, Occ, and Det risk ratings as the inputs, and it produces an RPN score, i.e., multiplying the Sev, Occ, and Det ratings [see (1)] to yield the output. The definitions of the input and output spaces are as follows.

Definition 1: An input space, i.e., Sev × Occ × Det, is considered. Variables sev, occ, and det are the elements of Sev, Occ, and Det, respectively, sev ∈ Sev, occ ∈ Occ, and det ∈ Det, respectively. The lower and upper bounds of Sev are represented by sev and sev, respectively. Similarly, the lower and upper bounds of Occ and Det are represented by occ and occ, and det and det, respectively. Usually, sev = occ = det = 1, and sev = occ = det = 10.

A set of data samples in the Sev, Occ, and Det space, as defined in Definition 1, i.e., \( \mathcal{X} = \{ \text{sev}_k, \text{occ}_k, \text{det}_k \} \), where \( k = 1, 2, 3, \ldots, n \), is considered. Traditionally, the risk of \( \text{sev}_k, \text{occ}_k, \text{det}_k \) is compared with those from other sets of data samples in the RPN space, as in Definition 2.

Definition 2: The RPN space is the output space containing all possible RPN scores, i.e., RPN ∈ RPN space. The lower and upper bounds of the RPN space are represented by RPN and RPN, respectively.

B. Conventional and FIS-Based RPN Models

Traditionally, the RPN is obtained using

\[
\text{RPN}_k = \text{sev}_k \times \text{occ}_k \times \text{det}_k. \tag{1}
\]

In an FIS_RPN model [4], [11], the product function in (1) is replaced by an FIS model. The FIS model is used to aggregate three risk factors, i.e., Sev, Occ, and Det, and produces an FIS_RPN score. In contrast to the conventional RPN model, each Sev, Occ, and Det rating is defined using a scale table with a number of partitions, as in Definition 3.

Definition 3: Each Sev, Occ, and Det domain is defined using a scale table, with \( m_{\text{sev}} \), \( m_{\text{occ}} \), and \( m_{\text{det}} \) partitions, respectively. Each partition is represented by a fuzzy membership function, i.e., \( \mu^{\text{sev}}(x) \), and is associated with a linguistic term, i.e., \( A^x \), where \( x = 1, 2, 3, \ldots, m_X, x \in \{ \text{sev, occ, det} \} \), and \( X \in \{ \text{sev, occ, Det} \} \).

An ordered sequence of \( \mu^X(x) \) exists, as in Definition 4. In this paper, the sufficient condition from the works in [11] and [12] is used.

Definition 4: The fuzzy membership function, i.e., \( \mu^X(x) \), follows an ordered sequence, i.e., \( \mu^{X+1}(x) \leq \mu^X(x), \) where

\[
p_X \in [1, 2, 3, \ldots, m_X - 1]. \note{This note is added to indicate the range of \( p_X \).}
\]

Note that \( \mu^{X+1}(x) \leq \mu^X(x) \) if Condition 1 of Theorem 1 (as in Section III-C) is satisfied.

In this paper, the zero-order Sugeno FIS model is chosen owing to its effectiveness in modeling the nonlinear relationship between the inputs and the output [25], as well as its universal approximation property [26]. Each fuzzy if–then rule is represented as follows.

\[
R^{n_{\text{sev}}, n_{\text{occ}}, n_{\text{det}}, \text{RPN}} \left( \text{sev}, \text{occ}, \text{det} \right) \leftarrow \text{RPN} = A^n_{\text{sev, occ, det}}, \text{ and Det} = A^n_{\text{det}}, \text{ then the RPN is } R^n_{\text{sev, occ, det}}, \text{ and Det} = A^n_{\text{det}}, \text{ where } R^n_{\text{sev, occ, det}} \text{ is the fuzzy consequence in the RPN space. Note that } R^n_{\text{sev, occ, det}} \text{ is the fuzzy singleton of } R^n_{\text{sev, occ, det}}. \text{ Using the zero-order Sugeno FIS model, the FIS_RPN score is obtained with (2) [11], shown at the bottom of the page, which is hereafter denoted as the classical FIS_RPN model. The lower and upper bounds for } R^n_{\text{sev, occ, det}} \text{ are denoted as } \alpha \text{ and } \beta, \text{ respectively.}
\]

C. Monotonicity Property

Based on the findings in [11] and [12], the fulfillment of the monotonicity property is determined according to Definition 5. A sequence, i.e., \( \mathcal{S} \), denotes a subset of \( \{ \text{sev, occ, det} \} \), where \( x \) is excluded, i.e., \( \mathcal{S} \subseteq \{ \text{sev, occ, det} \} \) and \( x \notin \mathcal{S} \) is considered.

Definition 5 [11]: The classical FIS_RPN model is said to fulfill the monotonicity property if the FIS_RPN score increases or remains unchanged as \( x \) increases, i.e., \( \text{FIS_RPN}(\mathcal{S}, x_2) \geq \text{FIS_RPN}(\mathcal{S}, x_1) \), where \( x_2 > x_1 \).

Theorem 1 [11]: The classical FIS_RPN model [i.e., (2)], is said to fulfill the monotonicity property between Sev, Occ, Det, and FIS_RPN if the following conditions are satisfied.

Condition 1: At the rule antecedent part, \( (d\mu^{p_x+1}(x)/dx)/\mu^{p_x+1}(x) \geq (d\mu^{p_x}(x)/dx)/\mu^{p_x}(x) \) Note that \( (d\mu^{p_x}(x)/dx)/\mu^{p_x}(x) \) is the ratio between the rate of change of the membership function and the membership function itself.

Condition 2: At the rule consequent part, \( b^{p_x+1, \mathcal{P}} \geq b^{p_x, \mathcal{P}} \), where \( p_x \in [1, 2, 3, \ldots, m_X - 1] \).

Condition 2 implies that the fuzzy rule base should be complete, i.e., all \( (m_{\text{sev}} \times m_{\text{occ}} \times m_{\text{det}}) \) fuzzy rules should be available, and Corollary 1, which is an extension of Condition 1, applies.

Corollary 1 [11]: Let \( \mu^n(x) \) be a Gaussian membership function, i.e., \( \mu^n(x) = e^{-[(x-c^n_s)^2/2\sigma^n_s]^2} \), where \( c^n_s \) and \( \sigma^n_s \) are the center and width of the membership function, respectively.

(1) The ratio \( (d\mu^{p_x}(x)/dx)/\mu^{p_x}(x) \) of a Gaussian membership function returns a linear function as follows:

\[
E^{p_x}(x) = -\frac{1}{\sigma^{p_x}_x} x_i + \frac{c^{p_x}_x}{\sigma^{p_x}_x^2}.
\]

(2) Condition 1 of Theorem 1 is satisfied if \( E^{p_x+1}(\mathcal{S}) \geq E^{p_x}(\mathcal{S}) \) and \( (E^{p_x+1}(\mathcal{S}) \geq E^{p_x}(\mathcal{S})) \) are valid.

\[
\text{FIS_RPN} \left( \text{sev, occ, det} \right) = \frac{\sum_{n_{\text{sev}}=1}^{m_{\text{sev}}} \sum_{n_{\text{occ}}=1}^{m_{\text{occ}}} \sum_{n_{\text{det}}=1}^{m_{\text{det}}} \left( \mu^{n_{\text{sev}}(\text{sev})} \times \mu^{n_{\text{occ}}(\text{occ})} \times \mu^{n_{\text{det}}(\text{det})} \times b^{n_{\text{sev}}, n_{\text{occ}}, n_{\text{det}}} \right) \times \mu^{n_{\text{sev}}(\text{sev})} \times \mu^{n_{\text{occ}}(\text{occ})} \times \mu^{n_{\text{det}}(\text{det})}}{\sum_{n_{\text{sev}}=1}^{m_{\text{sev}}} \sum_{n_{\text{occ}}=1}^{m_{\text{occ}}} \sum_{n_{\text{det}}=1}^{m_{\text{det}}} \left( \mu^{n_{\text{sev}}(\text{sev})} \times \mu^{n_{\text{occ}}(\text{occ})} \times \mu^{n_{\text{det}}(\text{det})} \right)} \tag{2}
\]
IV. PROPOSED METHOD AND ANALYSIS

A. Proposed Interval-Based Method

An antecedent, i.e., $A^\text{Sev}\land A^\text{Occ}\land A^\text{Det}$, where $A^\text{Sev}\in\mathcal{N}^\text{Sev}, A^\text{Occ}\in\mathcal{N}^\text{Occ}$, and $A^\text{Det}\in\mathcal{N}^\text{Det}$, is considered. The upper and lower regions of $A^\text{Sev}\land A^\text{Occ}\land A^\text{Det}$ are defined as follows.

**Definition 6:** The upper and lower regions of $A^\text{Sev}\land A^\text{Occ}\land A^\text{Det}$ are denoted as $A^\text{Sev}\land A^\text{Occ}\land A^\text{Det}$, respectively. Note that $A^\text{Sev}\land A^\text{Occ}\land A^\text{Det}$ consist of antecedents that satisfy $A^\text{Sev} \geq \text{Sev}, A^\text{Occ} \geq \text{Occ}, A^\text{Det} \geq \text{Det}$ and $A^\text{Sev} \leq \text{Sev}, A^\text{Occ} \leq \text{Occ}, A^\text{Det} \leq \text{Det}$, respectively.

Suppose a set of fuzzy rules from an FMEA user (a domain expert) is provided as follows.

**Definition 7:** A set of original fuzzy rules is denoted as $R^\text{Sev}\land\text{Occ}\land\text{Det}$, respectively. A set of original fuzzy rules is denoted as $R^\text{Sev}\land\text{Occ}\land\text{Det}$, respectively.

1. $R^\text{Sev}\land\text{Occ}\land\text{Det}$ is complete if all $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is provided.
2. $R^\text{Sev}\land\text{Occ}\land\text{Det}$ is incomplete if not all $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is provided.
3. $R^\text{Sev}\land\text{Occ}\land\text{Det}$ is monotone if Condition 2 of Theorem 1 is satisfied.
4. $R^\text{Sev}\land\text{Occ}\land\text{Det}$ is nonmonotone if Condition 2 of Theorem 1 is not satisfied. Noise is modeled in terms of judgment errors from FMEA users, leading to nonmonotone fuzzy rules.
5. The unknown $R^\text{Sev}\land\text{Occ}\land\text{Det}$ rules are denoted as $R^\text{Sev}\land\text{Occ}\land\text{Det}$.
6. $b^\text{Sev} \pm \text{Occ} \pm \text{Det}$ are always known.

In this paper, each consequent $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is transformed into a set of intervals denoted as $[b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}]$ in order to obtain an interval FIS_RPN model. Note that $b^\text{Sev}\land\text{Occ}\land\text{Det}$ are obtained in two steps. In the first step, $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is transformed into an interval using

$$\frac{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}}{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}} - \sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}} = \min(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$$

$$\frac{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}}{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}} - \sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}} = \max(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$$

In other words, the first step transforms $b^\text{Sev}\land\text{Occ}\land\text{Det}$ into a set of monotone intervals. If $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is monotone, i.e., satisfying Condition 2 of Theorem 1, $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always valid. Again, a sequence, i.e., $\pi$, denotes a subset of $\text{Sev}, \text{Occ}, \text{Det}$, where $x$ is excluded, i.e., $\pi \subset \text{Sev}, \text{Occ}, \text{Det}$: $x \notin \pi$ is considered.

**Theorem 2.1:** $b^\pi\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone, i.e., satisfying Condition 2 of Theorem 1.

**Proof:** $b^\pi\text{Sev}\land\text{Occ}\land\text{Det}$ is a subset of $b^\pi\text{Sev}\land\text{Occ}\land\text{Det}$. Using (3), $b^\pi\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone, and it satisfies Condition 2 of Theorem 1.

**Theorem 2.2:** $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone, i.e., satisfying Condition 2 of Theorem 1.

**Proof:** $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is a subset of $b^\text{Sev}\land\text{Occ}\land\text{Det}$, using (4), $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone, and it satisfies Condition 2 of Theorem 1.

**Theorem 2.3:** $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone, i.e., satisfying Condition 2 of Theorem 1.

**Proof:** From (3), the limit of $\min(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$ is $\min(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$. From (4), the limit of $\max(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$ is $\max(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$. Therefore, $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always true.

In the second step, $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is obtained using

$$\frac{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}}{\sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}} - \sum_{\text{Sev}}\sum_{\text{Occ}}\sum_{\text{Det}}} = \max(b^\text{Sev}\land\text{Occ}\land\text{Det} \in [b^\text{Sev}\land\text{Occ}\land\text{Det}, b^\text{Sev}\land\text{Occ}\land\text{Det}])$$

and $b^\text{Sev}\land\text{Occ}\land\text{Det}$ are always true. Therefore, $b^\text{Sev}\land\text{Occ}\land\text{Det}$ is always monotone.
Functions of Sev, Occ, and Det. These membership functions are designed in such a way that Corollary 1 is satisfied.

B. Measurement of Uncertainty

Uncertainty pertaining to the proposed interval FIS_RPN model can be measured by its size and coverage metrics, respectively, as in (9) and (10), shown at the bottom of the page. The size metric gives an indication of uncertainty in the RPN space with respect to a data sample in the input space (see Definitions 1–3).

The coverage metric is defined by a 4-D space of Sev × Occ × Det × RPN space (see Definitions 1 and 2) and is covered by the interval FIS_RPN model, as in (10). Note that if $R_{\text{Sev}}^{\text{Occ}} : A_{\text{sev}} \times A_{\text{occ}} \times A_{\text{det}} \rightarrow b_{\text{sev}} \times b_{\text{occ}} \times b_{\text{det}}$ is satisfied, Theorem 1, i.e., $FIS_{\text{RPN}}(\text{sev}, \text{occ}, \text{det}) = FIS_{\text{RPN}}(\text{sev}, \text{occ}, \text{det})$, is always true. Then, size$(\text{sev}, \text{occ}, \text{det}) = 0$ and coverage $= 0$ are always valid as well.

C. Risk Ordering

Two data samples, which are denoted as $\mathbf{x}_1 = \{\text{sev}_1, \text{occ}_1, \text{det}_1\}$ and $\mathbf{x}_2 = \{\text{sev}_2, \text{occ}_2, \text{det}_2\}$, are considered. With the proposed interval FIS_RPN model, $[FIS_{\text{RPN}}_1, FIS_{\text{RPN}}_2, FIS_{\text{RPN}}_3]$ and $[FIS_{\text{RPN}}_1, FIS_{\text{RPN}}_2]$ are obtained. A summary of the ordering between $\mathbf{x}_1$ and $\mathbf{x}_2$ is presented in Table I.

D. Proposed FMEA Procedure

In this paper, the proposed interval FIS_RPN model for handling incomplete and/or nonmonotone fuzzy rules is incorporated as part of the FMEA procedure. Fig. 5 shows the proposed FMEA procedure.

The details are as follows.

1) Start.

2) Develop the scale tables for Sev, Occ, and Det.

3) Design the membership functions for Sev, Occ, and Det.

Condition 1 of Theorem 1 is adopted to design the membership functions of Sev, Occ, and Det. These membership functions are designed in such a way that Corollary 1 is satisfied.

4) Gather experts’ knowledge in the fuzzy IF–THEN format.

5) Handle incomplete and/or nonmonotone fuzzy rules with the proposed two-stage reasoning scheme in Section IV-A.

6) Study the process or product and divide the process or product into subprocesses or subcomponents.

7) Determine all the potential failure modes of each process or component.

8) Determine the effects of each failure mode.

9) Determine the root causes of each failure mode.

10) List the current control or prevention action of each root cause.

11) Evaluate the impact of each effect (Sev ranking).

12) Evaluate the probability of Occ for each root cause (Occ ranking).

13) Evaluate the efficiency of the control or preventive actions (Det ranking).

14) Construct the interval $FIS_{\text{RPN}}$ model.

15) Order or prioritize the failure modes.

The proposed method in Section IV-C is adopted.

16) Correct any errors.

Return to Step 6 if there is any correction or refinement to be made.

17) End.
TABLE II
SCALE TABLE OF SEV

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Linguistic Term, (a_{\text{Sev}})</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None (Unnoticed)</td>
<td>• Unnoticed</td>
</tr>
<tr>
<td>2 – 5</td>
<td>Low (Special handling)</td>
<td>• Yield hit, cosmetic</td>
</tr>
<tr>
<td>6 – 7</td>
<td>Moderate (Quality/Convenience)</td>
<td>• Impacts customer yield</td>
</tr>
<tr>
<td>8 – 9</td>
<td>High (Reliability/reputation)</td>
<td>• Customer impact</td>
</tr>
<tr>
<td>10</td>
<td>Very High (Liability)</td>
<td>• Failures will affect safety or compliance to law</td>
</tr>
</tbody>
</table>

TABLE III
SCALE TABLE OF OCC

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Linguistic Term, (a_{\text{Occ}})</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Remote</td>
<td>• Once ever</td>
</tr>
<tr>
<td>2</td>
<td>Low</td>
<td>• Once/quarter</td>
</tr>
<tr>
<td>3</td>
<td>Low</td>
<td>• Once/month</td>
</tr>
<tr>
<td>4 – 6</td>
<td>Moderate</td>
<td>• Once/week, several/month</td>
</tr>
<tr>
<td>7 – 8</td>
<td>High</td>
<td>• Many/week, few/week</td>
</tr>
<tr>
<td>9 – 10</td>
<td>Very High</td>
<td>• Many/shift, many/day</td>
</tr>
</tbody>
</table>

TABLE IV
SCALE TABLE OF DET

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Linguistic Term, (a_{\text{Det}})</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2</td>
<td>Very high</td>
<td>• Prevent excursion from occurring</td>
</tr>
<tr>
<td>3 – 4</td>
<td>High</td>
<td>• Controls are able to detect within the same machine/module</td>
</tr>
<tr>
<td>5 – 6</td>
<td>Medium</td>
<td>• Controls are able to detect within the same functional area</td>
</tr>
<tr>
<td>7 – 8</td>
<td>Low</td>
<td>• Control may not detect excursion until reach next functional area</td>
</tr>
<tr>
<td>9</td>
<td>Very low</td>
<td>• Control probably will not detect</td>
</tr>
<tr>
<td>10</td>
<td>Extremely low</td>
<td>• No control available</td>
</tr>
</tbody>
</table>

V. CASE STUDY

A. Background

A real-world case study to evaluate the proposed FMEA procedure in Section IV-D is presented. The case study focused on a test handler process of Flip Clip Ball Grid Array (FCBGA) production in a semiconductor plant. The test handler process aimed to facilitate functional and quality tests of FCBGA products. FMEA was used to analyze and improve the test handler process. The scale tables of Sev, Occ, and Det used are summarized in Tables II–IV, respectively. In each scale table, column “Ranking” shows the score intervals. These intervals were tagged with linguistic terms, as in column “Linguistic Term \(a_{\text{Sev}}\),” where \(n_x = 1, 2, 3, \ldots, m_x\), and \(X \in \{\text{Sev, Occ, Det}\}\). There were \(m_X\) intervals for each Sev, Occ, and Det rating, respectively. Tables II–IV summarize the evaluation criteria for each interval. As an example, a score of 2–5 was assigned with the linguistic term of “Low” for Sev, i.e., \(A_{\text{Sev}}^2\), corresponding to “Yield hit, cosmetic,” as shown in Table II. In this paper, \(\text{RPN} = 1\), and \(\text{RPN} = 1000\). There were five membership functions in the RPN domain, i.e., “Low,” “Low Medium,” “Medium,” “High Medium,” and “High,” with the corresponding \(b\) values of 1, 250.75, 500.5, 750.25, and 1000, respectively.

A set of complete and monotone fuzzy rules (i.e., 180 fuzzy rules) was gathered through discussion with engineers in charge of the test handler process. The proposed model was assessed using the complete set of fuzzy rules. To evaluate the effectiveness of the proposed model with incomplete rule sets, 30% and 50% of the fuzzy rules were randomly selected for the experimental study. They were denoted as \(R_{\text{Sev, Occ, Det, #}}\), \(A_{\text{Sev, Occ, Det, #}}\), and \(\# \rightarrow b_{\text{Sev, Occ, Det, #}}\). In addition, uniformly distributed noise was randomly injected to \(R_{\text{Sev, Occ, Det, #}}\) to produce nonmonotone fuzzy rules. As such, 20% and 30% of \(R_{\text{Sev, Occ, Det, #}}\) were randomly selected for noise injection.

Equation (11), shown at the bottom of the page, was used to add uniformly distributed noise to \(R_{\text{Sev, Occ, Det, #}}\), with noise injection (i.e., 20% and 30%); 5) an incomplete and nonmonotone rule base containing 180 rules with noise injection; 3) an incomplete rule base containing 30% randomly selected fuzzy rules with no noise injection; 4) a complete but nonmonotone rule base containing 30% randomly selected fuzzy rules with no noise injection; 4) a complete rule base containing 30% randomly selected fuzzy rules with no noise injection; 3) an incomplete rule base containing 30% randomly selected fuzzy rules with no noise injection; 2) an incomplete rule base containing 30% randomly selected fuzzy rules with no noise injection; 1) a complete rule base containing 30% randomly selected fuzzy rules with no noise injection; and 6) an incomplete and nonmonotone rule base containing 30% of randomly selected fuzzy rules with noise injection (i.e., 20% and 30%).

B. Risk Evaluation Outcomes

Table V summarizes the risk ordering outcomes for experiments 1–3, whereas Table VI shows the outcomes for experiments 4–6, with 30% noise. In Tables V and VI, a total of 15 failure modes, i.e., \(\pi_k\), where \(z = 15\), are listed in column “Failure Mode.” Columns “Sev,” “Occ,” and “Det” show the respective risk ratings describing each failure mode. In Table V, the outcomes from the conventional RPN model are shown in column “RPN score.” As an example, failure mode “1” with Sev, Occ, and Det of 1, 2, and 1, respectively, is associated with an RPN score of 2.

Subcolumns “Classical FIS_RPN model” and “Interval FIS_RPN model” show the results from the classical FIS_RPN model and interval FIS_RPN models, respectively. Columns “FIS_RPN score” and “FIS_RPN ranking” indicate the outcome and ranking from the classical FIS_RPN model, respectively. For the interval FIS_RPN model, subcolumns “FIS_RPN” and “FIS_RPN” show the failure risk evaluation

\[
b_{\text{Sev, Occ, Det, #}} = \begin{cases} 1, & \text{if } b_{\text{Sev, Occ, Det, #}} \text{ selected} + (2 \times \text{rand} - 1) \times 249.75 < 1 \\ 1000, & \text{if } b_{\text{Sev, Occ, Det, #}} \text{ selected} + (2 \times \text{rand} - 1) \times 249.75 > 1000 \\ \end{cases}
\]

(11)
outcomes in terms of intervals [i.e., (7) and (8)], whereas the ranking of each failure mode in its interval is presented in subcolumn “FIS_RPN ranking.” Based on the same example, using the classical FIS_RPN model, failure mode “1” is tagged with an “FIS_RPN score” and an “FIS_RPN ranking” of 97 and 1, respectively. With the interval FIS_RPN model, it is tagged with 97, 97, and 1 for “FIS_RPN,” “FIS_RPN,” and “FIS_RPN ranking,” respectively. When a complete set of fuzzy rules with no noise (i.e., Experiment #1) is used, FIS_RPN = FIS_RPN = FIS_RPN is always true. This leads to the same ordering outcome for both classical and interval FIS_RPN models.

Using the classical FIS_RPN model, failure modes 11 and 12 (Sev, Occ, and Det of 2, 10, and 1, respectively, for failure mode 11 and Sev, Occ, and Det of 2, 10, and 4, respectively, for failure mode 12) are associated with the same FIS_RPN score of 996 for the complete rule base, as highlighted in Table V. According to the monotonicity property (i.e., Definition 5), failure mode 12 should have a higher or equal FIS_RPN score as compared with failure mode 11. In this case, the monotonicity property is fulfilled. Indeed, it is easy to maintain the monotonicity property when the fuzzy rules are complete and monotone (i.e., without noise), as suggested by Theorem 1. Using the proposed interval FIS_RPN model, the same ordering outcome is obtained.

A similar scenario is observed for complete and incomplete rule bases with noise injection, in which the classical FIS_RPN model produces nonmonotone fuzzy rules. The same example is considered. In Table VI, for the complete rule base with 30% noise (i.e., Experiment #5), the classical FIS_RPN model produces FIS_RPN scores of 995 and 992 for failure modes 11 and 12, respectively. With 50% fuzzy rules and 30% noise (i.e., Experiment #6), the FIS_RPN scores are 933 and 680, respectively. For 30% fuzzy rules and 30% noise (i.e., Experiment #7), the FIS_RPN scores are 873 and 662, respectively. In these cases, the monotonicity property is violated, whereby inappropriate fuzzy ranking outcomes are obtained, i.e., the risk of failure mode 12 is lower than that of failure mode 11.

Using the proposed interval FIS_RPN model, the risk intervals of failure modes 11 and 12 are [980, 996] and [980, 997], respectively, with 50% fuzzy rules (i.e., Experiment #2). Based on (9), the interval sizes are 16 (or 996–980) and 17 (or 997–980), respectively. For 30% fuzzy rules and 30% noise (i.e., Experiment #9), the risk intervals of [873, 1000] and [971, 1000] are obtained, respectively. Again, with (9), the sizes are 127 (or 1000–873) and 29 (or 1000–971), respectively. For another 30% fuzzy rules (i.e., Experiment #4), the risk intervals of [894, 996] and [909, 996] are obtained, respectively. Again, with (9), the sizes are 102 (or 996–894) and 87 (or 996–909), respectively. Therefore, it can be concluded that the risk of failure mode 11 is lower than that of failure mode 12 based on the outcomes of the proposed interval FIS_RPN model. As such, a rational ordering of the failure modes is produced. In short, although different FIS_RPN scores are obtained in different experiments (complete set or different incomplete sets of fuzzy rules), a rational risk ordering outcome can be obtained using the proposed interval FIS_RPN model.

A similar scenario is observed for complete and incomplete rule bases with noise injection, in which the classical FIS_RPN model produces nonmonotone fuzzy rules. The same example is considered. In Table VI, for the complete rule base with 30% noise (i.e., Experiment #5), the classical FIS_RPN model produces FIS_RPN scores of 995 and 992 for failure modes 11 and 12, respectively. The FIS_RPN scores are 933 and 680, respectively, with 50% fuzzy rules and 30% noise (i.e., Experiment #6). With 30% fuzzy rules and 30% noise.
Experiment #7 and Experiment #8), the FIS_RPN scores are 744 and 662, and 59 and 7, respectively. In short, the monotonicity property is violated, and irrational risk ordering outcomes are obtained using the classical FIS_RPN model, i.e., the risk for failure mode 11 is higher than that for failure mode 12.

However, this problem can be solved with the proposed interval FIS_RPN model. In Experiment #5, the risk score is [976, 996], with a size of 20 (or 996–976) for both failure modes. In Experiment #6, [750, 996] and [750, 997] are obtained. In Experiment #7, [873, 1000] and [971, 1000] are obtained, and in Experiment #8, [841, 996] and [870, 996] are obtained. In short, the proposed interval FIS_RPN model produces a rational ordering outcome, although different FIS_RPN scores are obtained in different experiments (complete set or different incomplete sets of fuzzy rules with random noise injection), i.e., the risk for failure mode 12 is higher than or equal to that for failure mode 11. These results are supported by Theorems 2–4 in Section IV-A.

C. Ordering of Failure Modes

From a practical perspective, it is important to order all the failure modes for risk evaluation. In Table V, using the classical FIS_RPN model, a rational ordering of 15 failure modes can be obtained in Experiment #1. It can be observed that \( F_1 < F_2 < F_3 < F_4 < F_5 < F_6 < F_7 < F_8 < F_9 < F_{10} < F_{11} < F_{12} < F_{13} < F_{14} < F_{15} \). The same result can be obtained with the proposed interval FIS_RPN model as well. In short, the proposed interval FIS_RPN model is able to handle complete and monotone fuzzy rules, and it produces the same ordering outcome as the classical FIS_RPN model.

However, the classical FIS_RPN model cannot produce monotone FIS_RPN scores for incomplete and/or nonmonotone fuzzy rules. This can be explained with Theorem 1, in which a complete and monotone fuzzy rule set is required for the classical FIS_RPN model to produce such ordering [11], [12]. As an example, in Table V, in Experiment #3 with 30% fuzzy rules, the ordering outcome of \( F_1 < F_2 < F_3 < F_4 < F_5 < F_6 < F_7 < F_8 < F_9 < F_{10} < F_{11} < F_{12} < F_{13} < F_{14} < F_{15} \) is obtained with the classical FIS_RPN model. This ordering outcome suggests that \( F_6 < F_7 \), but this is not true because \( F_6 = [1, 5, 1] \) and \( F_7 = [1, 7, 1] \). The monotonicity property indicates that \( F_6 \leq F_7 \). Using the interval FIS_RPN model, \( F_6 < F_7 < F_8 < F_9 < F_{10} < F_{11} \). The monotonicity property is satisfied.

In Table VI, in Experiment #8 with 30% fuzzy rules and 30% noise, the ordering outcome of \( F_1 < F_2 < F_3 < F_4 < F_5 < F_6 < F_7 < F_8 < F_9 < F_{10} < F_{11} < F_{12} < F_{13} < F_{14} < F_{15} \) is obtained. This ordering suggests that \( F_2 < F_3 \), which is not true because \( F_2 = [1, 2, 1] \) and \( F_3 = [1, 2, 3] \). The monotonicity property indicates that \( F_2 \leq F_3 \). Again, using the interval FIS_RPN model, \( F_1 < F_2 < F_3 < F_4 < F_5 < F_6 \). The monotonicity property is satisfied.

In summary, both the classical and interval FIS_RPN models produce the same ordering outcomes for the case of complete and monotone fuzzy rules. However, the classical FIS_RPN model is not able to produce a rational ordering in the case of incomplete and/or nonmonotone fuzzy rules. The proposed interval FIS_RPN model is able to overcome this problem by producing rational ordering outcomes, even with incomplete and/or nonmonotone fuzzy rules. These experimental results are in agreement with Theorems 2–4.

D. Coverage of Interval FIS_RPN Model

The coverage measure of the interval FIS_RPN model is analyzed. Using (10), the coverage measure in the input and output spaces, i.e., \( Sev \times Occ \times Det \times RPN space \), results in a total of 728 271 unit volume (or 9, 10-1 × 9, (i.e., 10-1) × 9 (i.e., 10-1) × 999 (i.e., 1000-1) = 728 271 is obtained, as shown in Table VII. For the complete fuzzy rule base without noise, the coverage measure of the interval FIS_RPN model is zero. In other words, the intervals of FIS_RPN and FIS_RPN overlap completely, resulting in a crisp value. This indicates that the interval FIS_RPN model properly operates with a complete and monotone fuzzy rule base. When 50% and 30% fuzzy rules are used, the resulting coverage metric increases in general. When a higher percentage of noise is injected to the fuzzy rule base, the coverage measure increases.

Note that the coverage measure indicates the uncertainty in the input and output spaces resulting from the incomplete and nonmonotone fuzzy rules. It gives a measurement pertaining to the size of a certain area covered by the interval FIS_RPN model. Table VII summarizes the experimental results. For the complete fuzzy rule base without noise, the result (i.e., zero unit volume) implies that there is no area of uncertainty. With 50% fuzzy rules, 46 624 out of 728 271 unit volume (or 6.40% of the entire input and output space) indicate uncertainty. The resulting interval FIS_RPN model can be still useful and effective in practice as the coverage measure is relatively small. With 30% fuzzy rules and 30% noise, 118 660 out of 728 271 unit volume (or 16.29% of the entire input and output space) indicate uncertainty. In other words, the resulting interval FIS_RPN model has a relatively large uncertain space when it operates with 30% fuzzy rules and 30% noise.

E. Monotonicity Test

The monotonicity property of the classical and interval FIS_RPN models, which is denoted as \( FIS_RPN = f(sev, occ, det) \) and \( FIS_RPN \times FIS_RPN = f(sev, occ, det) \), respectively, can be evaluated using the monotonicity test in [11]. The test provides a metric to indicate whether an FIS_RPN model is monotone. Specifically, two comparable sets of risk factors are compared using

\[
monotone(x) = \begin{cases} 
1, & f(\overline{x}, x + 1) \geq f(\overline{x}, x) \\
0, & \text{else}
\end{cases}
\]  

(12)

where \( x \subseteq [sev, occ, det] \). Sequence \( \overline{x} = [s_1, s_2] \) denotes a subset of \([sev, occ, det] \), where \( x \) is excluded. For the interval
FIS_RPN model, both FIS_RPN and FIS_RPN are evaluated separately. The FIS_RPN, FIS_RPN, and FIS_RPN scores are rounded to integers. The following equation shows that the monotonicity test index resides in the range between 0 and 900 (i.e., \(9 \times 9 \times 10 = 900\)) for all possible combinations of \(s\) with respect to \(x\):

\[
\text{Monotonicity Index}(x) = \sum_{s_2=1}^{9} \sum_{s_1=1}^{10} \sum_{x=1}^{9} (\text{monotone}(x)).
\]

(13)

The monotonicity test index of all three risk factors is obtained by considering \(x = \text{sev}, \text{occ}, \text{det}\), i.e.,

\[
\text{Monotonicity test index = Monotonicity Index(sev)} + \text{Monotonicity Index(occ)} + \text{Monotonicity Index(det)}.
\]

(14)

As such, a range between 0 and 2700 \((3 \times 900 = 2700)\) is resulted. If the monotonicity test index is 2700, the monotonicity property of the FIS_RPN model is preserved. Similarly, FIS_RPN and FIS_RPN are separately evaluated for the interval FIS_RPN model. To evaluate the stability of the proposed approach, five experiments with random selection of 30% and 50% of fuzzy rules with noise are conducted. If the monotonicity test index is smaller than 2700, the monotonicity property is violated.

Table VIII summarizes the monotonicity test results of different classical and interval FIS_RPN models using (14). Subrow “configuration” is the experimental setup, e.g., “complete fuzzy rules with no noise.” No random selection is involved for complete fuzzy rules without noise. As such, the test is conducted only once, and it is labeled as A in subrow “Experiment.” Subrows “FIS_RPN,” “FIS_RPN,” and “FIS_RPN” show the monotonicity test results. For the case of “complete fuzzy rules with no noise,” the monotonicity property is preserved (indicated as monotonicity preserved (MP) in Table VIII) when the monotonicity test index is 2700. For other configurations, e.g., 50% fuzzy rules without noise, a series of experiments with five different random selections has been conducted, which are labeled as “A,” “B,” “C,” “D,” and “E.”

For the complete fuzzy rule base without noise, the monotonicity property is preserved for FIS_RPN, FIS_RPN, and FIS_RPN. This is in agreement with Theorems 1 and 4, which indicate that, when Conditions 1 and 2 are satisfied, a monotone ordered model is produced. Fig. 6 illustrates the graphical plots of the FIS_RPN scores versus Sev and Occ at Det = 3 using the classical FIS_RPN model with a complete fuzzy rule base without noise. A monotone surface is shown in Fig. 6.

However, this is not the case for 50% and 30% fuzzy rule bases with and without noise. For 50% and 30% fuzzy rules without noise, the monotonicity test index scores of the classical FIS_RPN model are 1457 and 1425 (Experiment A), respectively. The problem can be solved with the interval FIS_RPN model, whereby the same score of 2700 (MP) for both FIS_RPN and FIS_RPN is obtained. Figs. 7 and 8 are the graphical plots of the FIS_RPN scores versus Sev and Occ at Det = 3 using the classical and interval FIS_RPN models with 50% fuzzy rules (Experiment A), respectively. A nonmonotone surface is shown in Fig. 6, whereas a pair of monotone interval surfaces is produced by the interval FIS_RPN model, as shown in Fig. 8. The experiments with five different random selections of fuzzy rules suggest that the interval FIS_RPN model can preserve the monotonicity property. The same observation can be made with other configurations of 20% and 30% noise. With the interval FIS_RPN model, the monotonicity property is achieved in all five different random selections of fuzzy rules.
A similar scenario can be also observed for other cases of incomplete fuzzy rule bases with and without noise. For 50% fuzzy rules and 30% noise, the classical FIS_RPN model produces a monotonicity test score of 1455 (Experiment A). Again, the problem can be solved using the interval FIS_RPN model, whereby a score of 2700 (i.e., MP) is obtained for both FIS_RPN and FIS_RPN. Figs. 9 and 10 illustrate the surface plots of FIS_RPN scores versus Sev and Occ at Det = 3 for the classical and interval FIS_RPN models, respectively. A nonmonotone curve is observed in Fig. 9, whereas a pair of monotone interval surfaces is shown in Fig. 10.

In Table VIII, the classical FIS_RPN model fails to preserve the monotonicity property in all cases. This observation is in line with Theorem 1, which indicates that a complete and monotone fuzzy rule base is required. In addition, it can be observed that, for incomplete fuzzy rule bases with and without noise, the interval FIS_RPN model can achieve monotonicity (i.e., converge). This can be explained with Theorems 2–4.

Remarks

In this paper, the monotonicity test score [i.e., (14)] is used to measure the degree of fulfillment of the monotonicity property by FIS_RPN models. If the monotonicity test score is 2700, the monotonicity property is preserved, e.g., see Fig. 6. Otherwise, the monotonicity property is violated, e.g., see Figs. 7 and 9. It is also possible to use the monotonicity test score as a measure of monotonicity pertaining to FIS_RPN models. Two FIS_RPN models with monotonicity test scores of 1433 and 2491, as in Figs. 11 and 12, respectively, are considered. It can be observed that the model with a score of 2491 is more monotone than that with a score of 1433.
VI. CONCLUSION AND FUTURE WORK

In this paper, an analytical interval FIS_RPN model has been proposed to overcome a key limitation of the classical FIS_RPN model, i.e., the requirement of complete and monotone fuzzy rules. A method to order the failure modes in conjunction with the proposed FIS_RPN model has been also introduced. An FMEA case study using real information obtained from a semiconductor manufacturing plant has been conducted. The results demonstrate the effectiveness of the interval FIS_RPN model in handling scenarios with incomplete and/or nonmonotone fuzzy rules. In addition, the interval FIS_RPN model allows all failure modes to be appropriately ordered, even when the provided fuzzy rules are incomplete and/or nonmonotone. The proposed approach is important as it constitutes a new and effective monotone fuzzy reasoning scheme applicable to a variety of industrial applications and designs using FMEA [5–8].

For further research, the proposed interval FIS_RPN model will be generalized for application to other domains, e.g., decision-making problems, education assessments, and risk assessment. In addition, other methods to transform the interval-valued outputs to crisp outputs, which include entropy and Jacard similarity, will be examined. The use of the monotonicity test to measure the “monotonicity-ness” concept [27] of FIS-based models will be also investigated. An index for measuring the monotonicity of functions [27] to determine which functions in a given class are more monotone than those from other classes will be studied.

REFERENCES


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